Dynamics of the Transient Negative Eddy Response to Zonal-Mean Zonal
Wind Variations

David J. Lorenz

\textsuperscript{a} Center for Climatic Research, University of Wisconsin-Madison, Madison, Wisconsin

\textit{Corresponding author:} David J. Lorenz, dlorenz@wisc.edu
ABSTRACT: Many studies have focused on the long-term positive feedback between Annular Mode zonal wind (U) perturbations and the eddy momentum fluxes (M). Lagged correlation analysis between U and M anomalies, however, shows that a transient period of negative eddy forcing follows the peak in zonal wind anomalies. This negative forcing is more ubiquitous than the positive feedback because it occurs for all U EOFs not just EOF1. It has been hypothesized that this response is either (1) an intrinsic feature of the eddies independent of the U or (2) caused by U induced changes in Rossby wave reflection. Here it is shown that the response can be reproduced in a GCM by imposing a rapid change in U, and therefore mechanism (1) does not appear to be relevant. Furthermore, the transient response can be generated in a model when there are no turning latitudes, and therefore mechanism (2) does not appear to be relevant. Instead it is shown that the transient response is due to the adjustment of a preexisting eddy field to a change in the background wind. This transient effect is negative when the meridional scale of the U change is small enough compared to the waves, and vice versa. The sign of the initial response depends on the relative size of advection by U versus retrogression by the background vorticity gradient on the meridional tilt of the Rossby waves. Finally it is shown that this transient response has a large damping effect on U variability.
1. Introduction

The internal variability of the zonal-mean zonal wind ($\bar{u}$) is driven by stochastic anomalies in the zonal-mean eddy momentum flux convergence (eddy forcing). Therefore lagged correlations between eddy forcing and $\bar{u}$ show the strongest correlations when the eddy forcing leads $\bar{u}$ by a few days. This relationship is expected from the zonal-mean momentum budget. In many ways, the most interesting relationship is at lags where $\bar{u}$ leads the eddy forcing. For EOF1, Lorenz and Hartmann (2001) and Lorenz and Hartmann (2003) found positive correlations at large lags where $\bar{u}$ leads the eddies and they argued that this is evidence of a positive feedback between $\bar{u}$ and the eddy forcing.

Much less attention has been paid to the transient period of negative eddy forcing that follows 5-7 days after the peak in $\bar{u}$ anomalies. This transient negative eddy forcing is much more prevalent than the positive feedback because it occurs in response to all $\bar{u}$ EOFs not just EOF1. Lorenz and Hartmann (2001) argued that this negative feature was inherent to the eddies and was independent of the $\bar{u}$ anomalies. Zurita-Gotor et al. (2014) argued that the negative eddy forcing was due to the boundedness of wave activity. Lorenz (2015), however, showed through initial value experiments that the transient eddy forcing was in fact a response to $\bar{u}$ anomalies, and therefore this transient eddy forcing is a negative feedback on $\bar{u}$ variability. Rivière et al. (2016) and Robert et al. (2017) argued that wave reflection is responsible for this negative feedback. That the negative eddy forcing is a wave-mean-flow feedback was also discovered much earlier by Robinson (1994), who performed GCM experiments where periodic EOF1 $\bar{u}$ anomalies were imposed in the model. For low frequencies the eddies reinforced the $\bar{u}$ anomalies while for high frequencies the eddies damped the $\bar{u}$ anomalies.

In this paper we propose an alternate explanation for the transient negative feedback and provide a quantitative theory for its impacts on $\bar{u}$ variability and externally forced change. We begin with a description of the GCMs used in this study and then look at the internal variability of the GCMs. Next, the role of the zonal-mean on the transient response is shown via initial value experiments with imposed EOF $\bar{u}$ anomalies. This is followed by GCM experiments where the zonal-mean state is fixed at a constant value. The eddy forcing variability in this simulation provides a quantitative picture of the impact of the transient negative feedback on the eddy forcing autocorrelation and power spectrum. Next we develop a quantitative analytic model of the transient feedback based
on the non-divergent barotropic vorticity equation. This model captures the zonal wavenumber
dependence of the transient feedback and its independence on details of the mean state. The model
shows that the transient response is due to the adjustment of a pre-existing eddy field to a change
in background $\bar{u}$. Finally, we extend the feedback analysis of Lorenz and Hartmann (2001) and
Lubis and Hassanzadeh (2021) to include the transient feedback and discuss the implications for
variability and forced change.

2. Methods

a. GCMs

The multi-level GCM is a standard primitive equation spectral model integrating the vorticity,
divergence, temperature and the log surface pressure. The sigma coordinate vertical differencing
scheme of Simmons and Burridge (1981) is used. An $8^{th}$ order hyperdiffusion with a time scale of
0.1 days for the smallest scale waves is applied to the model variables. The only non-typical aspect
of the model is the time differencing, which is the AB3-AI2 method of Durran and Blossey (2012).
The resolution of all simulations is T42 with 20 equally spaced vertical levels. The multi-level
control simulation is forced with the diabatic heating and frictional damping of Held and Suarez
(1994). The model is run for 6500 days and the first 500 days are discarded to allow for model
spin-up.

For the simulations with fixed zonal-mean, we use a two-level primitive equation model (GCM)
based on Hendon and Hartmann (1985). The prognostic variables of vorticity, divergence and
potential temperature are defined on two pressure levels (250 and 750mb). In the equations, the
vertically integrated divergence is constrained to be zero for consistency with the assumption of
constant surface pressure at the lower boundary. For thermal forcing we use Held and Suarez
(1994) and for mechanical damping we use Rayleigh friction at the lower-level with an $e$-folding
time scale of 3 days. This is denoted the control simulation for the two-level model. In the past,
this model has been run at coarse resolution (R15) with semi-implicit time differencing (Hendon
and Hartmann (1985); Robinson (1991)). However, at the T42 resolution used here, the speed of
the winds is approximately equal to the phase speed of the fastest gravity mode (which is slower
than usual due to the constant surface pressure) and therefore our version is fully explicit with third
order Adams-Bashforth time differencing (Durran (1991)). The model is run for 6500 days and the first 500 days are discarded to allow for model spin-up.

### 3. Results

#### a. Internal Variability

The time mean zonal-mean zonal winds (U) with Held and Suarez (1994) forcing has a mid-latitude jet centered at about 43° latitude (Fig. 1a). Next, the EOFs of the instantaneous vertical- and zonal-mean zonal wind are calculated (the vertical average is from 1000 to 100mb) and then the $\bar{u}$ anomalies are regressed on the resulting PCs (Fig. 1bc). EOF1 has oppositely signed center of actions on either side of the jet maximum and therefore represents north/south shifts in the mid-latitude jet. EOF2 is a tripolar pattern representing a strengthening and narrowing of the jet in its positive phase. The relationship between EOF1 and EOF2 and the mean state are consistent with Southern Hemisphere observations (Lorenz and Hartmann (2001)).

The PC1 and PC2 autocorrelations (Fig. 2a) demonstrate that EOF1 is much more persistent than EOF2. The decay of the autocorrelation in time is not uniform: for the first 5-7 days the autocorrelation drops more rapidly than at longer time lags. The lagged correlation between the PCs and the vertically-averaged (1000-100mb) eddy momentum flux convergence, or eddy forcing (Lorenz and Hartmann (2001)), projected onto the corresponding EOF pattern is shown in Fig. 2b. At negative lags, the eddy forcing leads the $\bar{u}$ anomalies and these correlations include the stochastic eddy fluxes that force the $\bar{u}$ anomalies in the first place. At large positive lags, the response of the eddy momentum fluxes to the $\bar{u}$ anomalies can be seen, which is positive for EOF1 and zero for EOF2 (Lorenz and Hartmann (2001)). Most studies ignore the transient period of negative/reduced eddy forcing at short positive time lags (up to 5-7 days). This feature is responsible for the rapid drop off in autocorrelation at short lags noted above. This transient negative response also occurs for all higher order EOFs (not shown). Lorenz and Hartmann (2001) hypothesized that this negative eddy forcing is an intrinsic feature of the eddies, although they offered no specific mechanism. Zurita-Gotor et al. (2014) also believed that the transient negative forcing is intrinsic to the eddies and argued that the cause is the boundedness of wave activity. Lorenz (2015), on the other hand, show through initial value experiments that the transient forcing is a response to zonal-mean anomalies and therefore involves wave-mean-flow interactions rather than wave-wave...
1. a) Time- and zonal-mean zonal wind ($\bar{u}$) from the multi-level GCM (m/s). b) $\bar{u}$ anomalies regressed on PC1 of instantaneous vertical-average $\bar{u}$ variability (m/s) c) Same as (b) but for PC2.

Fig. 1. a) Time- and zonal-mean zonal wind ($\bar{u}$) from the multi-level GCM (m/s). b) $\bar{u}$ anomalies regressed on PC1 of instantaneous vertical-average $\bar{u}$ variability (m/s) c) Same as (b) but for PC2.

interactions. To explain the transient feedback, Rivière et al. (2016) and Robert et al. (2017) implicated a negative U/eddy forcing feedback involving wave reflection on the equatorward flank of the jet. Note that the lagged correlations show that this feedback is not simply proportional to $\bar{u}$, because otherwise the lagged correlations at positive lags would have the same shape as the
Fig. 2. a) Autocorrelation of PC1 and PC2 of \( \bar{u} \) from the multi-level GCM. b) Lagged correlation between PC1 and PC2 of \( \bar{u} \) and the eddy forcing (= vertical-average (1000-100mb) eddy momentum flux convergence projected on the corresponding EOF of vertical average \( \bar{u} \)). Positive lags mean the \( \bar{u} \) anomalies lead the eddy forcing. c) Same as (b) but for the eddy forcing by zonal wavenumber 4 alone. d) Same as (b) but for the eddy forcing by zonal wavenumber 8.

PC autocorrelations. All of the above features are evident in Northern and Southern Hemisphere observations (Lorenz and Hartmann (2001); Lorenz and Hartmann (2003)).

The transient negative forcing is primarily due to longer zonal wavenumbers. For example, for zonal wavenumber 4 (Fig. 2c) the negative forcing is about as large as the previous positive forcing. For zonal wavenumber 8 (Fig. 2d), on the other hand, there is no transient reduction in forcing at all and for EOF2 the transient effect is very weak. When looking at wavenumber 4, one also sees a quasi-periodic element to the transient forcing that continues to oscillate until lags 30 to 40 days. Apparently, the oscillations of individual wavenumbers destructively interfere past the initial negative forcing so that the total transient forcing rapidly decays to zero past 7 days.

b. Initial Value Experiments

Here we repeat the initial value experiments of Lorenz (2015), which “branch off” the long control simulation of the multi-level GCM described in Section 2a (Fig. 3). The eddy field of each
Fig. 3. Schematic of the initial value experiments or “branch simulations” used in this section. The purple line represents the evolution of the long control simulation in time. The initial value experiments branch off of the control simulation every 8 hours. The eddy fields of each initial value experiment are exactly the same as the control simulation at the time of the branch. The zonal-mean state at the start of the initial value experiment, on the other hand, is the time- and zonal-mean $u$, temperature ($T$) and surface pressure ($p_s$) plus or minus the $u$, $T$ and $p_s$ associated with EOF1 or EOF2 of $\bar{u}$. The branch simulation is exactly the same as the control simulation at that time. The zonal-mean state, on the other hand, is suddenly switched to a prescribed $u$, temperature and surface pressure state at the start of the initial value experiment. Afterwards the initial value experiments evolve freely for 30 days with the same Held and Suarez (1994) forcing of the control simulation. We perform an initial value experiment for all 18000 eddy fields that are archived for the 6000 day control run (i.e. the GCM data was saved every 8 hours). The entire series of 18000 initial value experiments is performed four times for four different zonal-mean states. The zonal-mean states consist of the time and zonal-mean $u$, temperature ($T$) and surface pressure ($p_s$) plus or minus the zonal-mean $u$, $T$ and $p_s$ associated with EOF1 or EOF2 of $\bar{u}$. Next we average the eddy forcing\(^1\) over 18000 initial value experiments and both hemispheres, which gives the mean eddy forcing response as a function of latitude and time (0-30 days). The response to EOF1 is defined as the difference between the mean eddy forcing response to the positive EOF1 state and the negative EOF1 state. Because the eddy field of the positive and negative EOF state is exactly the same at time = 0, the eddy forcing

\(^1\)The eddy forcing is defined to be the vertical- (1000-100mb) and zonal-average eddy momentum flux convergence.
Fig. 4. Lagged regression of eddy forcing onto the normalized PC1 of $\overline{u}$ (red). Positive lags mean the $\overline{u}$ anomalies lead the eddy forcing. The eddy forcing response to EOF1 perturbations in the initial value problems (described in text) projected onto the EOF1 structure (blue line). b) Same as (a) but for EOF2/PC2.

difference is identically zero at time = 0. Afterwards, the eddy field evolves differently in response to the different zonal-mean states. The EOF2 response is defined in the same way. In Fig. 4a, the mean eddy forcing response to EOF1 is projected back onto the EOF1 pattern to get the positive feedback of the eddy forcing as a function of time (blue curve). This blue curve means that 2-3 days after EOF1 is “switched on”, the eddies strongly act to damp the $\overline{u}$ anomalies. By 7 days after the switch on, however, the eddies reinforce the $\overline{u}$ anomalies. This blue curve is analogous to the positive lags of a lagged regression of the EOF1 eddy forcing with $\overline{u}$ PC1 (red curve in Fig. 4a). The time scale of the transient forcing and the eventual long-term positive eddy response are remarkably similar between the initial value experiments and the lagged regression. The only difference is that the transient negative forcing is much larger for the initial value experiment. The same exercise is repeated for EOF2 and the agreement between initial value and regression in this case is much better. An explanation of the transient response amplitude discrepancy is given in Section 4d.
The transient eddy forcing appears to be a response to the \( \bar{u} \) anomaly because the only difference in the initial conditions is in the zonal-mean—the initial eddy fields are identical. Also, the transient response in the above experiments is not due to nonlinearities in the eddy response to \( \bar{u} \) anomalies. For example, if we halve the initial \( \bar{u} \) amplitude the response is the same except half as small (not shown). The response is also not due to subtle changes in \( \bar{u} \) structure by day 7 which then lead to a reversal of the eddy response. For example, we have reinitialized the \( \bar{u} \) every 24 hours of the initial value simulation back to the original \( \bar{u} \) anomaly and the character of the response does not change at all (not shown, the only change is that the long-term response is slightly bigger because the \( \bar{u} \) anomaly is not allowed to decay with time). The above experiments strongly suggest that the transient negative response is inherently transient and linear, and that it is a response to the \( \bar{u} \) anomaly. Therefore the wave-wave mechanism of Zurita-Gotor et al. (2014) does not appear to be operating.

In Fig. 5, the eddy forcing response from Fig. 4 is divided into the contributions of the different zonal wavenumbers. The left panels show the initial value experiments and the right panels show the lagged regression response for positive lags only. Consistent with Fig. 2, the long zonal wavenumbers are responsible for the transient response. For EOF1, wavenumbers 4 and 5 are most responsible while for EOF2 wavenumber 5 is most responsible. The transient response also exists for wavenumbers longer than 4, however, in this GCM the amplitude of these waves is weak so they do not contribute much to the negative response. Overall the responses for the initial value and lagged regression experiments are similar, however, as noted above the initial negative response is stronger in the initial value experiments. In addition, it appears that the long-term positive response of the shorter waves for EOF1 takes some time (5-7 days) to develop in Fig. 4a. In the lagged regression, on the other hand, the \( \bar{u} \) anomalies, on average, have existed for some time prior to lag 0 and therefore the positive long term response exists from the beginning of Fig. 4b.

Watterson (2002) found similar transient negative responses in experiments with a barotropic model. Motivated by this study, we perform a large number of initial value experiments like the previous paragraph but in a non-divergent barotropic model. To convert the multi-vertical-level fields of the GCM to a single level we perform a weighted vertical average using the EOFs of the eddy streamfunction as described in Lorenz (2015). The barotropic model is inviscid except for hyperdiffusion, which is the same as the GCM. The response of the eddy forcing projected on the
Fig. 5. a) The eddy forcing response to EOF1 perturbations in the initial value problems (described in text) projected onto the EOF1 structure for each individual zonal wavenumber from 1 to 12. b) Same as (a) but for the lagged regression of eddy forcing on PC1 from the internal variability for positive lags only ($\bar{u}$ leads eddy forcing). c) Same as (a) but for EOF2. d) Same as (b) but for EOF2/PC2.

corresponding EOF structure for our barotropic experiments is shown in Fig. 6ab in red. The analogous response in the GCM is also shown in blue. The transient response is very well captured in the barotropic model for both EOF1 and EOF2. The only missing response is the long-term positive feedback for EOF1, which is explained by the lack of baroclinic instability to maintain the amplitude of the eddy field in the barotropic model. Note this lack of long-term feedback does not imply the “baroclinic feedback” is operating because the baroclinic feedback states that anomalies in baroclinic instability are positively correlated in latitude with anomalies in $\bar{u}$ and eddy forcing (Robinson (2000)). In our case, on the other hand, the lack of long-term feedback is due to the complete lack of baroclinic instability anywhere. These barotropic experiments are still useful for understanding the transient feedback because it involves the long waves, which do not decay rapidly. For example, the Eddy Kinetic Energy (EKE) of zonal wavenumber 4 decays with an
Fig. 6. The eddy forcing response to EOF1 perturbations in the GCM initial value problems (blue) and the barotropic model initial value problems (red) projected onto the EOF1 structure. b) Same as (a) but for EOF2. c) Same as (a) except the barotropic model is linearized about the a basic state with $\bar{u} = 0$. d) Same as (c) but for EOF2. e) Same as (c) expect the barotropic model uses the approximation described in (4). f) Same as (e) but for EOF2.

The $e$-folding time scale of 13 days (not shown). The long-term feedback, on the other hand, involves short waves and the EKE of zonal wavenumbers greater than five decay rapidly with an $e$-folding time scale of 3 days in these barotropic experiments (not shown). The rapid decay of the short waves is energetically consistent with the fact that the momentum flux of these waves reinforces the mean jet.

To eliminate potential mechanisms for the transient feedback, we repeat the barotropic experiments but now 1) the eddy dynamics is linearized and 2) the background $\bar{u}$ is set to zero instead of the time-mean state from the GCM. So, in other words, these experiments the eddies are linearized
about either the positive or negative phase of the appropriate EOF. The eddy response in these
experiments (Fig. 6cd) is almost unchanged from before. Because the turning latitudes of the
eddies depend critically on the mean jet, this experiment suggests that the reflection mechanism of
Rivière et al. (2016) and Robert et al. (2017) is not responsible for the transient negative response.
In section 4 we develop an analytic model that explains the transient response.

\textit{c. Fixed Zonal Mean}

The model experiments so far have all involved initial value problems, but the transient feedback
has a profound impact on the long-term internal variability as well. In this Section, we will quantify
the impact of the transient feedback on $\bar{u}$ and eddy forcing autocorrelation and power spectrum.
To this end, we perform GCM simulations with a fixed zonal-mean state. For the remainder of
this paper, we use the two-level GCM (see section 2a) because the eddy fluxes under a fixed zonal-
mean state are close to the standard control simulation. The control $\bar{u}$ in the freely running (i.e.
zonal-mean is not fixed) two-level model is given by the blue lines in Fig. 7a. Like the multi-level
GCM (Fig. 1a), the mean jet is at 43°. The EOFs of the vertical average $\bar{u}$ anomalies (Fig. 7b)
are also similar in structure to the multi-level GCM. The lagged correlations between the $\bar{u}$ PCs
and the eddy forcing (Fig. 7c) agree with the multi-level GCM (Fig. 2b) except that the positive
feedback (correlations at large positive lags) is weaker for the two-level model.

For the fixed zonal-mean simulation, the most obvious choice of zonal-mean state is that of the
time-mean of the control simulation, however, the global mean EKE, vertical EP flux (Edmon et al.
(1980)) and eddy momentum flux increase by 35%, 28% and 22%, respectively, when this zonal-
mean state is used (not shown). It appears that zonal-mean fluctuations in the control simulation
limit eddy activity via a negative feedback between bursts of eddy heat flux and zonal-mean
baroclinicity. This elevated eddy amplitude makes comparisons between the control simulation
and the fixed zonal-mean problematic. To create fixed zonal-mean state that better captures the
coupling between the zonal-mean and eddies, we first run a simulation like the control but where
all zonal-mean time tendencies are reduced by a factor of $S$. The specific value of $S$ is does not
affect our conclusions and we have tried reducing the tendencies by factors of 10 to 500. A value
of $S = 100$, is a good compromise between damping short-term zonal-mean fluctuations while
still allowing the model to equilibrate in a reasonable length of time (we spin-up the model for
Fig. 7. a) Time- and zonal-mean zonal wind for the two-level model control simulation (blue). The upper level is a solid line and the lower level is a dashed line. The same fields but for the slow-zonal two-level model (see text) are shown in orange. b) Vertical-average $\bar{u}$ anomalies regressed on PC1 (blue) and PC2 (red) of instantaneous vertical-average $\bar{u}$ variability (m/s). c) Lagged correlation between PC1 and PC2 of $\bar{u}$ and the corresponding eddy forcing. Positive lags mean the $\bar{u}$ anomalies lead the eddy forcing.

10000 days before archiving data when $S = 100$). Allowing the eddy field to set its “preferred“ nearly-fixed zonal-mean state leads to a slight decrease in zonal-mean vertical wind shear (4%) and slight changes in low level $\bar{u}$ in the polar regions (Fig. 7a). Next we perform a simulation with the zonal-mean fixed to the slow-zonal-mean time-mean state. The increases in eddy statistics in this fixed zonal-mean simulation are much smaller: the global mean EKE, vertical EP flux and eddy
Fig. 8. a) The autocorrelation of the eddy forcing of EOF1 for the two level model. The two-level control simulation is blue and the fixed-zonal-mean simulation (see text) is orange. b) Same as (a) but for EOF2. c) Lagged correlation between the eddy forcing of EOF1 and the eddy forcing of EOF2. The control simulation is blue and the fixed-zonal-mean simulation (see text) is orange. Positive lags mean EOF1 leads EOF2.

momentum flux increase by 6%, 2% and 4%, respectively (not shown). This new fixed zonal-mean simulation is used for all results below.

A comparison between the fixed-zonal-mean and standard control simulation is shown in Fig. 8. The statistics used here are based on the eddy forcing projected on either the EOF1 or EOF2 structures from the control simulation. For EOF1 eddy forcing, the control simulation autocorrelation dips below zero at short lags and the becomes slightly positive at longer lags. These features are
consistent with the transient negative response and positive feedback, respectively, seen in Fig. 7c. When $\bar{u}$ is fixed, the positive feedback disappears and the transient feedback almost disappears and its time scale increases. For EOF2, the transient feedback completely disappears when $\bar{u}$ is fixed (Fig. 8b).

The cross correlation between the EOF1 and EOF2 eddy forcing shows that negative EOF2 leads to positive EOF1 and positive EOF1 leads to positive EOF2. This relationship weakly extends to larger lags in a quasi-periodic manner. This relationship means that eddy forcing anomalies propagate equatorward in latitude at small time scales. This contrasts with longer time scales where poleward propagation dominates (e.g. James and Dodd (1996); Feldstein (1998); Lee et al. (2007); Chemke and Kaspi (2015); Sheshadri and Plumb (2017)). This difference between high and low frequency eddy behavior was first noted in Sparrow et al. (2009). Unlike the autocorrelations, the cross correlation is not impacted much by the fixed zonal-mean. This suggests that the equatorward propagation in the eddy field is not due to wave-mean flow interaction. A logical explanation for this eddy forcing behavior is the equatorward propagation of wave packets from the mid-latitudes where baroclinic instability in greatest, to their critical levels in the subtropics. Because of the strong bias toward equatorward wave propagation on a sphere, the cross correlation is not isotropic but instead is biased in the same sense. The equatorward propagation of wave packets may also be the source of the small negative correlations in the EOF1 autocorrelation for fixed $\bar{u}$ (Fig. 8a): because the EOF1 centers of action straddle the baroclinic source of wave packets, the wave packet propagation signal is more evident in this pattern than for EOF2, which has side lobes more distant from the jet core.

The eddy forcing power spectra for the fixed-zonal-mean and control run are shown by the solid lines in Fig. 9. The spectra have a relative/global maxima at frequencies around 0.14 day$^{-1}$. From this maxima, the power decreases at lower frequencies until, for EOF1 alone, the long-term positive feedback leads to increased power at the lowest resolved frequencies. The pronounced minima around 0.05 day$^{-1}$ for EOF1 and at the lowest resolved frequencies for EOF2 is unusual for a geophysical time series. This unusual structure in the eddy forcing power spectrum is synonymous with the transient negative feedback due to 1) the mathematical relationship between the power spectrum and auto-covariance and 2) the strong relationship between eddy forcing and $\bar{u}$ from the momentum budget. Zurita-Gotor et al. (2014) emphasized this unusual peak in the eddy forcing
Ma et al. (2017) also concluded that this feature is independent of the zonal-mean and further argued that the eddy forcing spectrum should linearly decay to zero at the lowest frequencies in the absence of wave-mean-flow interactions. However, the power spectra in fixed-zonal-mean simulations (solid orange line) clearly show that the eddy forcing power increases as frequency
goes to zero in the absence of wave-mean-flow interactions. The only minor exception is a small relative maxima for EOF1 at 0.08 day$^{-1}$, which is associated with the small dip below zero in the EOF1 autocorrelation (Fig. 8a). We argued above that this is due to the equatorward propagation of Rossby wave packets that are localized in latitude.

In addition to eddy forcing, quasi-periodicity is observed in the eddy heat fluxes and eddy kinetic energy associated with the Baroclinic Annular Mode (BAM, Thompson and Woodworth (2014)). Thompson and Barnes (2014) argue that the periodicity comes from negative feedbacks between baroclinicity and eddy heat fluxes. Zurita-Gotor (2017), however, showed that the phase difference between heat flux and baroclinicity are not consistent with this hypothesis. Here we show that the quasi-periodicity in eddy heat flux is remarkably similar to the eddy forcing and therefore the periodicity in BAM might also involve the adjustment of a preexisting eddy field to background flow transience. First we show the eddy heat flux power spectrum in the two-level model (solid blue, Fig. 9c). At low frequencies (below 0.07 days$^{-1}$), there is a pronounced reduction in power, which is synonymous with a quasi-periodic peak at 0.07 days$^{-1}$. As noted by Zurita-Gotor (2017), calculating the power spectrum individually for each zonal wavenumber and then summing over all wavenumbers gives a curve with no peak (blue dotted line). Evidently, the reduction in eddy heat flux at the lowest frequencies is due to anti-correlation between the heat flux at a wavenumber $k_1$ and another wavenumber, $k_2$, rather than the reduction of the heat flux by the individual waves (Zurita-Gotor (2017)). The sum of the eddy forcing spectra over individual wavenumbers shows the same structure as the heat flux with no well-defined quasi-periodicity (dashed blue lines in Fig. 9ab). The eddy forcing and heat flux are also similar in that the quasi-periodicity disappears when the zonal-mean state is fixed (solid orange in Fig. 9abc). Moreover, the anti-correlation between different wavenumbers changes to positive correlation for all cases as demonstrated by the fact that the solid orange line is always larger than the dashed orange line at low frequencies.

From the above results, we conclude that wave-mean-flow feedbacks are responsible for the quasi-periodicity because it disappears when the zonal-mean is fixed. Furthermore, wave-mean-flow feedbacks are also responsible for the anti-correlation between waves. We believe that stochastic eddy forcing/heat flux by a given wavenumber makes zonal-mean anomalies that suppress other wavenumbers via transient feedbacks. Unlike the theory of Thompson and Barnes (2014), which involves the zonal-mean baroclinicity, the transient feedbacks considered here involve generic
wind anomalies that are not necessarily the same as the anomalies that most impact baroclinicity.

The positive correlation between different wavenumbers for the fixed-zonal-mean simulations might come about because eddy fluxes are naturally localized in longitude rather than in zonal wavenumber space, which leads to positively correlated zonal wavenumber fluxes. In future work, the impacts of zonal-mean transience on eddy heat fluxes and eddy kinetic energy will be studied in more detail.

4. Analytic Model of Transient Feedback

a. General Case

Subject to simplifying assumptions, we develop an analytic model of the transient feedback. First, consider a general linear model that represents the “typical” eddy state, $\zeta'$:

$$\frac{\partial \zeta'}{\partial t} + L(\zeta') = F', \quad (1)$$

where $\zeta'$ the eddy relative vorticity which is a function of $x$, $y$ and $t$, $L$ is a damped linear operator and $F$ is forcing. In our case, $L$ encapsulates the effect of the background zonal-mean flow on the eddies. If the linear operator is perturbed ($L \Rightarrow L + \hat{L}$), then the solution will also be altered ($\zeta' \Rightarrow \zeta' + \hat{\zeta}'$):

$$\frac{\partial \zeta' + \hat{\zeta}'}{\partial t} + L(\zeta' + \hat{\zeta}') + \hat{L}(\zeta' + \hat{\zeta}') = F'. \quad (2)$$

Subtracting (1) from (2), and expanding:

$$\frac{\partial \hat{\zeta}'}{\partial t} + L(\hat{\zeta}') + \hat{L}(\hat{\zeta}') + \hat{\zeta} = 0. \quad (3)$$

While there is no guarantee that $\hat{L}(\hat{\zeta}')$ is small compared to the other terms, we neglect this “double perturbation” term below. In this case it makes sense to rearrange (3):

$$\frac{\partial \hat{\zeta}'}{\partial t} + L(\hat{\zeta}') = -\hat{L}(\hat{\zeta}'), \quad (4)$$

which is now an inhomogeneous linear system for the unknown $\hat{\zeta}'$ given the “typical” eddy state ($\zeta'$) and the perturbed background flow ($\hat{L}$). To test the validity of the approximation in (4), we
repeat the linearized barotropic initial value problems in Section 3b but under the approximation in (4). Because the evolution of $\hat{\zeta}'$ in (4) involves $\zeta'$, we must also use the $\zeta'$ field from an integration of (1). In other words, both (4) and (1) must be integrated to obtain the solution to (4). Our integration of (4) is based on the final and simplest simulation in Section 3b where the background $\bar{u}$ is set to zero. Therefore, $L$ is the barotropic linear operator for no background flow, $\zeta'$ is the eddy vorticity from (1) and $\hat{L}$ is the change in the linear operator from either the positive or negative phase of the EOF pattern. The eddy momentum flux response from this new simulation (Fig. 6ef) is almost unchanged from the other initial value experiments (Fig. 6abcd), which is implies that neglecting $\hat{L}(\hat{\zeta}')$ is a reasonable approximation.

\[ b. \text{Non-divergent Barotropic Model} \]

We now apply (4) to a non-divergent barotropic equation in cartesian geometry $(x, y)$ linearized about a background zonal wind, $\bar{u}$, and vorticity gradient, $\beta - \bar{u}_{yy}$:

\[
\frac{\partial \zeta'}{\partial t} + \bar{u} \frac{\partial \zeta'}{\partial x} + (\beta - \bar{u}_{yy}) \frac{\partial \nabla^{-2} \zeta'}{\partial x} + D \zeta' = F',
\]

(5)

where $\zeta'$ is the eddy relative vorticity, $\partial \nabla^{-2} \zeta'/\partial x$ is the eddy meridional velocity ($v'$), $D$ is the Rayleigh damping coefficient, $F'$ is the eddy forcing, $\beta$ is the planetary vorticity and $\bar{u}_{yy}$ is the second derivative of $\bar{u}$ with respect to $y$.

We now make several simplifying assumptions in order to derive an analytic result: let the “basic-state” eddy forcing, $F'$, be proportional to $\exp(i k x)$, where $k$ is the zonal wavenumber. Next, let the unperturbed background state be uniform in $x$, $y$ and $t$ with a zonal-mean zonal wind, $U - c$, and background vorticity gradient, $\beta$. The zonal wind is denoted $U - c$ so that we account for non-zero basic-state eddy phase speeds, $c$ (i.e. the derivation is performed after a Galilean transform so that the forcing is stationary). For a uniform background state, the linear operator that corresponds to $L$ in (1) is:

\[
L(\zeta') = (U - c) \frac{\partial \zeta'}{\partial x} + \beta \frac{\partial \nabla^{-2} \zeta'}{\partial x} + D \zeta'.
\]

(6)
Because of the uniform background flow, the eddy vorticity is the same form as the forcing:

\[
\zeta' = \zeta \exp(ikx),
\]

(7)

where \( \zeta \) is a complex constant. For the derivation below it is not necessarily to relate \( \zeta \) to the forcing explicitly because (4) does not explicitly contain the forcing.

The perturbed operator, \( \hat{L} \), in (4) is

\[
\hat{L}(\zeta') = \Delta \bar{u} \frac{\partial \zeta'}{\partial x} - \Delta \bar{u}_{yy} \frac{\partial \nabla^{-2} \zeta'}{\partial x},
\]

(8)

where \( \Delta \bar{u} \) is the zonal-mean zonal wind anomaly. Note that there is no dissipation in this operator because \( \hat{L} \) is defined as the change in the \( L \) operator and the form of the dissipation does not change. To help understand the evolution of the (4), lets first consider the initial \( \hat{\zeta}' \) tendency after we suddenly turn on the \( \bar{u} \) anomaly from the initial condition \( \hat{\zeta}' = 0 \). This corresponds to the start of the initial value experiments in Section 3b. At this initial time, \( t = 0 \), (4) becomes:

\[
\frac{\partial \hat{\zeta}'}{\partial t} + \Delta \bar{u} \frac{\partial \zeta'}{\partial x} - \Delta \bar{u}_{yy} \frac{\partial \nabla^{-2} \zeta'}{\partial x} = 0.
\]

(9)

By definition, the total vorticity field, \( Z' \), is \( \zeta' + \hat{\zeta}' \). In addition, \( \hat{\zeta}' \) does not depend on space at \( t = 0 \) and \( \zeta' \) does not depend on time (see (7)), and therefore the full vorticity also obeys (9) at \( t = 0 \):

\[
\frac{\partial Z'}{\partial t} + \Delta \bar{u} \frac{\partial Z'}{\partial x} - \Delta \bar{u}_{yy} \frac{\partial \nabla^{-2} Z'}{\partial x} = 0,
\]

(10)

which has the exact same form as the original vorticity equation, (5) but without forcing and dissipation. Advection, which is the second term in (10), will act to tilt lines of constant phase in the direction of the imposed \( \bar{u} \) anomaly. By the kinematics of a non-divergent flow, phase lines tilted in this way are synonymous with eddy momentum fluxes that reinforce the imposed \( \bar{u} \) anomaly or, in other words, a positive feedback. The third term in (10) is the Rossby wave retrogression term, which is associated with anomalies in the vorticity gradient: \( -\bar{u}_{yy} \). To quantify the impact of Rossby wave regression, we use the fact that \( \nabla^{-2} Z = -Z' / k^2 \) at \( t = 0 \) and assume the
perturbed background flow takes a simple sinusoidal form in $y$:

$$\Delta \bar{u} = u(t) \cos(ny), \quad (11)$$

with amplitude $u(t)$, which depends on time, and meridional wavenumber, $n$. The temporal dependence of $\Delta \bar{u}$ is not important until later. In this case $\Delta \bar{u}_{yy} = -n^2 \Delta \bar{u}$ and (10) becomes:

$$\frac{\partial Z'}{\partial t} + \Delta \bar{u} \frac{\partial Z'}{\partial x} - \frac{n^2}{k^2} \Delta \bar{u} \frac{\partial Z'}{\partial x} = 0. \quad (12)$$

The initial tendency from the retrogression term opposes the advection and hence it is a negative feedback on $\Delta \bar{u}$. For long waves, $k$ is small and $n^2/k^2 > 1$ and therefore the negative feedback from retrogression is dominant. For short waves, $k$ is large and $n^2/k^2 < 1$ and therefore the positive feedback from advection is dominant. The dominance of the retrogression effect for small $k$ is consistent with the fact that the transient negative eddy forcing is caused by the long waves. The theory also predicts that the transient eddy forcing is positive for short waves. There is a small hint of this behavior at very short lags for zonal wavenumbers 8 and 9 in Fig. 5a. In Appendix A, we solve (4) for the barotropic vorticity equation for all times (i.e. not just $t = 0$) under the simplifying assumptions introduced in this section. In appendix A, we also find the eddy forcing (i.e. eddy momentum flux convergence) anomaly produced by this solution. By solving the system in detail, we see that the theory correctly predicts that the damping long wave response should be an order of magnitude bigger than the reinforcing short wave response. The solution is discussed in the next section.

c. Understanding the Solution

In Appendix A, we derive the following equation satisfied by the eddy forcing due to the transient feedback:

$$\frac{d^2m}{dt^2} + 2D \frac{dm}{dt} + \left(\omega^2 + D^2\right) m = \frac{n^2(k^2-n^2)}{k^2(k^2+n^2)} \frac{\partial}{\partial t} \left(\frac{du}{dt} + Du\right) \quad (13)$$
where $m$ is the anomalous eddy forcing acting on the imposed sinusoidal $\bar{u}$ anomaly (for simplicity, we choose not to use $\hat{m}$ for the perturbation eddy forcing),

$$\omega = k \left( U - c - \frac{\beta}{k^2 + n^2} \right), \quad (14)$$

$u$ is the time varying amplitude of the $\bar{u}$ anomaly and $\bar{\zeta}^2$ is the variance of the background eddy field (i.e. $\bar{\zeta}^2$ is a constant). Under the assumptions above, there is no eddy forcing out-of-phase with $\bar{u}$ in latitude. The homogeneous solution to (13) is a damped oscillator:

$$m = e^{-D t} \left( a \cos(\omega t) + b \sin(\omega t) \right), \quad (15)$$

where $a$ and $b$ are constants. The frequency of the oscillation, (14), is the difference between the frequency of the basic-state wave ($kc$) and the frequency of a free Rossby wave with scale $k^2 + n^2$.

In the GCM/observations there are a wide range of different frequencies and spatial scales, which leads to destructive interfere of the transient response at long time scales (see below). Hence, the transient response is localized at short time lags. For individual wavenumbers, however, one can see hints of the damped oscillations (Fig. 2c). Note that (13) does not include the long-term positive feedback because it is derived from a state with uniform background winds and no baroclinic instability. Theories of the positive feedback involve either the configuration of reflecting/critical levels in a latitudinally-varying mean state (Lorenz (2014)) or changes in the source of wave activity from baroclinic instability (Robinson (2000)).

To understand the dominance of the long waves on the transient response one must consider the forcing on the right hand side of (13). The forcing is proportional to the eddy vorticity variance ($\bar{\zeta}^2$) in the time mean, so the response is larger for zonal wavenumbers with larger vorticity variance. The response is also proportional to a factor, which we denote $P$:

$$P = \frac{n^2(k^2 - n^2)}{k^2(k^2 + n^2)}, \quad (16)$$

To estimate $n$, the spatial scale of the $\bar{u}$ EOF, we note that the main centers of action EOF1 are located at $36^\circ$ and $55^\circ$ degrees (Fig. 1a). This corresponds to half a wavelength. To convert to the equivalent zonal wavelength at the jet center ($\sim 45^\circ$), one can straightforwardly derive the following
a) zonal wavenumber dependence of forcing in analytic model

Fig. 10. The blue line is the variance of the eddy relative vorticity as a function of zonal wavenumber at 43° averaged from 500 to 100mb in the multi-level GCM. The red line is the blue line multiplied by the wavenumber dependent factor in the transient $m$ forcing equation (i.e. multiplied by $P$, (16)). The orange line is the blue line multiplied by $Q$, (18).

The calculated value of $n = 6.7$,

\[
 n = \frac{180 \cos(45^\circ)}{55^\circ - 36^\circ} = 6.7, \tag{17}
\]

which has units of number of waves around latitude circle. The calculated value of $n (= 6.7)$, which is the dividing point between positive and negative transient response, agrees well with the wavenumber dependence of the transient response in Fig. 5a. EOF2 has a slightly smaller spatial scale and the negative response extends to higher zonal wavenumbers (Fig. 5c).

Equation (16) also captures the weak positive transient response for high wavenumbers. First we show the eddy vorticity variance, $\zeta'^2$, at the center of the jet at upper levels (average from 500 to 100mb) for different zonal wavenumbers (blue in Fig. 10). The vorticity variance peaks at wavenumber 5 and decays more rapidly on the low wavenumber side of the maximum. Multiplying vorticity variance by the $P$ factor (16) to convert to the transient forcing (red line), we see that the negative forcing by the low wavenumbers strongly dominates over the high wavenumbers. In fact, the low wavenumber response show peak at zonal wavenumbers 1 and 2, which is not supported by the initial value experiments (Fig. 5ac). This discrepancy might be due to the fact that the amplitude of wavenumbers 1 and 2 strongly peak in the polar regions (not shown) and therefore these waves have a more significant meridional wavenumber, $l$, than the higher wavenumbers. This conflicts with the $l = 0$ assumption of the analytic model. Nevertheless, the analytic model clearly
reproduces the weak transient response for high wavenumbers. To understand, the source of the wavenumber dependence, we alternatively multiply the vorticity variance by:

\[ Q = \frac{k}{n} \left( 1 - \frac{n^2}{k^2} \right), \]  

which is the factor in the forcing of the perturbed vorticity equation (A5) scaled by the constant \( n \) to make it dimensionless. This factor does not explain the relative amplitude of the low and high wavenumbers (orange line in Fig. 10), therefore the amplitude dependence comes from the transformation from \( \zeta' \) to \( \zeta'\nu' \). The fact that the meridional wind in \( \zeta'\nu' \) emphasizes longer waves than the vorticity plays a role (contributes a \( \sim 1/k \) factor). However, the biggest contributor involves subtle issues when calculating the in-phase component of \( \zeta' \) and \( \nu' \) when going from (A12) to (A13) in Appendix A (contributes a \( \sim 1/k^2 \) factor).

The forcing of the eddy forcing, \( m \), in (13) also includes the interesting expression \( du/dt + Du \). If the dissipation for the waves, \( D \), is the same as the dissipation for the zonal-mean, which is a reasonable approximation, then \( du/dt + Du \) is the same as the eddy momentum flux convergence via the vertical average momentum budget (e.g. Lorenz and Hartmann (2001)). This eddy momentum flux convergence includes \( m \) itself, as well as the stochastic and long-term feedback component of the eddy forcing. We have explored separating the components by putting the \( m \) portion of \( du/dt + Du \) on the left hand side with the other \( m \) terms. However, the biggest contribution to the forcing in (13) is the stochastic component of the eddy forcing, hence it is not unreasonable to understand the system in its current form.

The \( du/dt + Du \) expression in (13) also implies the transient forcing is zero when a \( \bar{u} \) anomaly is decaying via Rayleigh friction. For example, at large positive lags, the eddy forcing for EOF2 is zero (Fig. 2b and Fig. 7c), which implies \( du/dt + Du = 0 \) and therefore the transient forcing is zero. If the transient forcing depended on either \( du/dt \) or \( Du \) alone, the \( m \) response would persist onwards past short positive lags. For example, if the forcing were proportional to \( -du/dt \) then the initial negative response would be followed by a positive response as the EOF2 anomaly decayed (\( du/dt < 0 \) implies positive transient response by the long waves). For EOF1, which decays slower than the Rayleigh damping rate due to the positive long-term feedback, \( du/dt + Du > 0 \) and therefore the transient feedback is partially offsetting the positive feedback at large positive lags.
In this section, we consider the impact of the super-position of a range of waves of different $k$ and $c$ on the solution to the transient response equation, (13). First, we solve (13) in terms of an integral using the method of variation of parameters:

$$m(t) = \int_{-\infty}^{t} P\bar{\zeta}^2 \left( \frac{du(s)}{dt} + Du(s) \right) \frac{\sin(\omega(t-s))}{\omega} \exp(-D(t-s)) \, ds,$$  \hspace{1cm} (19)$$

where, as before, $P$ is a constant defined by (16) and $\bar{\zeta}^2$ is the (constant) background eddy amplitude. Consistent with (13), (19) gives a damped oscillator response to a sudden impulse of the forcing: $du/dt + Du$. In the initial value experiments, on the other hand, the transient response to a sudden impulse is of the form $t \exp(-\alpha t)$, where $\alpha$ is a constant (not shown precisely, but see Fig. 4 and 6). This discrepancy can be resolved by noting that (19) applies to a single wave of zonal wavenumber, $k$ and phase speed, $c$ ($\omega$ is a function of $k$ and $c$, see (14)). In reality, there are a wide range of different waves in the GCM and therefore (19) must be integrated over $k$ and $c$:

$$m(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{t} P\bar{\zeta}^2 \left( \frac{du(s)}{dt} + Du \right) \frac{\sin(\omega(t-s))}{\omega} \exp(-D(t-s)) \, ds \, dk \, dc.$$  \hspace{1cm} (20)$$

Switching the order of integration and taking terms independent of $c$ and $k$ outside the interior integration, we get

$$m(t) = \int_{-\infty}^{t} \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P\bar{\zeta}^2 \frac{\sin(\omega(t-s))}{\omega} \, dk \, dc \right] \left( \frac{du}{dt} + Du \right) \exp(-D(t-s)) \, ds.$$  \hspace{1cm} (21)$$

Note that $\bar{\zeta}^2$ is a function of $k$ and $c$ (for example, from a phase speed/latitude spectrum (Randel and Held (1991))) and $P$ is a function of $k$ (16). Given the approximations in the analytic model, calculating the double integral inside the brackets from the phase speed/latitude spectrum of $\bar{\zeta}^2$ does not give an impulse response function of the required form (i.e. $t \exp(-\alpha t)$). In particular, the transient response is contaminated from an excessively large response in zonal wavenumbers 1 and 2. We believe this error due to the assumption that the background waves are independent of $y$ because the barotropic models in Fig. 6 are able to capture the observed response even under the approximations in (4).
To go forward, we note that the quantity in brackets in (21) is a function of $t - s$. If we assume this function is of the form $t \exp(-\alpha (t-s))$, then the $m(t)$ response to a sudden impulse is of the desired form $t \exp(-\alpha t)$. Under this assumption

$$m(t) = A \int_{-\infty}^{t} (t-s) \exp(-\alpha (t-s)) \left( \frac{d u(s)}{d t} + D u(s) \right) ds,$$

where $A$ is a constant that represents the amplitude of the transient feedback. While the functional shape of the impulse response function in (22) is not derived from fundamentals, the $du/dt + Du$ forcing in (22) is analytically derived. In the next section we use (22) in a generalized feedback analysis that includes the transient feedback as well as the positive and poleward propagating feedbacks (Lubis and Hassanzadeh (2021)).

Equation (22) also explains why the transient feedback is bigger in the initial value experiment compared to the lagged regression for EOF1 (Fig. 4a). In the initial problem (blue line in Fig. 4a), $du/dt + Du$ is a delta function at $t = 0$ so all the forcing of the transient feedback occurs at $t = 0$. For the lagged regression (red line), one should first note that, via the momentum budget, the eddy forcing equals $du/dt + Du$ and therefore the plotted lagged regression is proportional to the lagged regression between $\bar{u}$ and the forcing of the transient feedback. So, in the lagged regression, the forcing of transient feedback is spread out over a wide range of negative lags and hence much of the transient response has long since dissipated by the time $t > 0$. Unlike EOF1, most of the EOF2 lagged regression (red line in Fig. 4b), is restricted to small negative lags (note that the area of the red line from lags -5 to 0 is almost as much as the integral over all negative lags). In this case, the initial value and lagged regression transient response are similar because the $du/dt + Du$ forcing occurs close to lag 0 in each.

e. Feedback Analysis

Unlike previous feedback analyses, which assume the eddies react instantaneously to $\bar{u}$ anomalies, the analytic model above shows that the transient eddy response is a weighted average of prior $\bar{u}$ anomalies (22). In the absence of additional information, we assume the same prior weighting for the positive and poleward propagating feedbacks. The feedback model includes coupling between EOF1 and EOF2 for the long-term feedbacks (Lubis and Hassanzadeh (2021)) but for the transient feedback we assume no coupling. This is consistent with the small change in eddy forcing cross
correlation between EOF1 and EOF2 when $\bar{u}$ is fixed to climatology (Fig. 8c). Under the above assumptions, the eddy forcing for EOF1 can be written:

$$m_1(t) = \tilde{m}_1(t) + a_1 \sum_{s=0}^{\infty} w(s) d_1(t-s) + b_{11} \sum_{s=0}^{\infty} w(s) u_1(t-s) + b_{12} \sum_{s=0}^{\infty} w(s) u_2(t-s),$$

(23)

where $m$ is the total eddy forcing, $\tilde{m}$ is the random eddy forcing unrelated to the $\bar{u}$ anomalies, $t$ is the time, $u$ is the $\bar{u}$ principal component, $d = du/dt + Du$, $w(s)$ is the discretized weighting based on (22) except normalized

$$w(s) = \alpha^2 s \exp(-\alpha s) \Delta t,$$

(24)

$\Delta t$ is the temporal resolution of the data, the subscripts refer to the EOF index (i.e. EOF1 or EOF2) and the integrals are discretized. The Rayleigh damping, $D$, which is assumed to be identical for the waves and the zonal-mean, is estimated as in Lorenz and Hartmann (2001) and is 0.1343 day$^{-1}$. The constant $a_1$ is the amplitude of the transient feedback for EOF1 and $b_{11}$ and $b_{12}$ are the feedback of EOF1 and EOF2 $\bar{u}$ anomalies onto EOF1, respectively (i.e. positive feedback and poleward propagation, see Lubis and Hassanzadeh (2021)). Next, we multiply (23) by $u_1(t-l)$, where $l$ is a time lag, and then average over time, which converts products such as $u_j(t)u_k(t-l)$ to lagged covariances:

$$M_{11}(l) = a_1 \sum_{s=0}^{\infty} w(s) D_{11}(l-s) + b_{11} \sum_{s=0}^{\infty} w(s) U_{11}(l-s) + b_{12} \sum_{s=0}^{\infty} w(s) U_{21}(l-s),$$

(25)

where $M_{jk}(l)$ denotes the lagged covariance between $m_j$ and $u_k$ at lag $l$ (positive $l$ means $u$ leads $m$), $D$ is the same but for $d$ and $u$, $U$ is the same but for $u$ and $u$, and we assume the lag, $l$, is positive and large enough such that $\tilde{m}$ is uncorrelated with $u$ (so the $\tilde{m}$ term is zero). Multiplying (23) by $u_2(t-l)$, gives a similar equation

$$M_{12}(l) = a_1 \sum_{s=0}^{\infty} w(s) D_{12}(l-s) + b_{11} \sum_{s=0}^{\infty} w(s) U_{12}(l-s) + b_{12} \sum_{s=0}^{\infty} w(s) U_{22}(l-s).$$

(26)

Equations (25) and (26) without the first term on the right are the same as Lubis and Hassanzadeh (2021) except for the weighting over prior lags with $w(s)$. In this Lubis and Hassanzadeh (2021) case, there are two equations for two unknowns and the solution is straightforward. In our case,
the additional unknown requires more information, which we obtain by fitting multiple time lags together at once. Because the system is now over-determined, we choose parameters $a_1$, $b_{11}$ and $b_{12}$ that minimize the sum over lags of the squared difference of the left and right hand sides of (25) and (26). To obtain the equations for $a_2$, $b_{22}$ and $b_{21}$, simply replace all 1 subscripts with 2 and all 2 subscripts with 1. The parameter $\alpha$, which is the temporal scale of the weights, is found by trial and error until the value that minimizes the squared error is obtained. The details are given in Appendix B.

The feedback analysis is applied to the two-level primitive equation model because we can check the accuracy of the feedback parameters estimated on the control simulation with the fixed-zonal-mean simulation. In future work, we will apply the analysis to observations. Unlike all prior feedback analyses, which consider only the long-term feedback and therefore use time lags greater than a week, we use all time lags from day 2 to day 40 for fitting the parameters. The estimated transient feedback parameters are:

$$
\begin{pmatrix}
D a_1 \\
D a_2
\end{pmatrix} = 
\begin{pmatrix}
-0.1059 \\
-0.1131
\end{pmatrix},
$$

(27)

where we multiply by $D$ so the units are days$^{-1}$ like the long-term feedback. The transient feedback parameters, $a_1$ and $a_2$, are negative as expected and moreover the amplitudes are nearly the same (6% difference). The estimated long-term feedback parameters are:

$$
\begin{pmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{pmatrix} = 
\begin{pmatrix}
0.1478 & 0.0667 \\
-0.0572 & -0.0049
\end{pmatrix},
$$

(28)

The parameter $\alpha$ in the temporal weighting function is 0.35 days$^{-1}$. The signs of the long-term feedback parameters, $b_{jk}$, are consistent with Lubis and Hassanzadeh (2021), except for $b_{22}$, which is very close to zero in both their and our analysis and therefore its sign is less robust. Given the sign convention of our EOFs (Fig. 7b), the signs of $b_{12}$ and $b_{21}$ imply the poleward propagation of $\bar{u}$ anomalies, which is consistent with multiple prior studies (e.g. James and Dodd (1996); Feldstein (1998); Lee et al. (2007); Chemke and Kaspi (2015); Sheshadri and Plumb (2017)).
Fig. 11. a) The autocorrelation of PC1 (solid blue) and PC2 (solid red) of the vertically averaged $\bar{u}$ in the two-level model. The dotted lines are the corresponding autocorrelation of the synthetic PC1 and PC2 time series generated from the transient/long-term feedback model (see text). b) Same as (a) but for the lagged correlation between a PC $\bar{u}$ and its eddy forcing. c) The auto-covariance of PC1 (solid blue) and PC2 (solid red) of the vertically averaged $\bar{u}$ in the two-level model. The dotted lines are the corresponding auto-covariance of the synthetic PC1 and PC2 time series with all feedbacks removed (see text). d) Same as (c) but for the lagged covariance.

Next, (23) and the analogous equation for $m_2(t)$ (replace all 1 subscripts with 2 and all 2 subscripts with 1) are used to create a “synthetic” time series of $u$ and $m$ (we use trapezoidal time differencing). For the eddy forcing without feedbacks, $\tilde{m}_j$, we use the time series from the GCM simulation with fixed zonal-mean state. The synthetic time series do a good job reproducing the autocorrelations of EOF1 and EOF2 and the lagged correlations between $\bar{u}$ anomalies and eddy forcing for both EOF1 and EOF2 (Fig. 11ab). Next we create a second synthetic time series with no transient and long-term feedbacks. The auto-covariances show that without feedbacks, the temporal structure of EOF1 and EOF2 are essentially identical to each other (dotted lines in Fig. 11c). Moreover, the variance (i.e. value at lag 0) is larger with no feedbacks. This is even the case for EOF1, which means the effect of the negative transient effect is larger than the positive feedback. For EOF1, the main effect of feedbacks is an increase in the persistence of EOF1, which is due
to the positive long-term feedback. In Fig. 11d, the “no feedback” lagged covariance between $\bar{u}$ and eddy forcing rise exponentially at negative lags and then abruptly drop to zero at positive lags. This shape is the same as that between red noise its white noise forcing. All prior analyses that remove feedbacks do not revert to this basic form, instead a transient negative response remains (e.g. Lorenz and Hartmann (2001)).

The fact that the transient response depends on $u$ via the term $du/dt + Du$ (19) means that the transient eddy feedback impacts the mean response to an external forcing. In other words, the term “transient feedback” is a misnomer as far as the response to external forcing. For variability, on the other hand, the $du/dt$ and $Du$ in the $m$ forcing cancel for positive lags (EOF2 and higher order EOFs) or nearly cancel (EOF1), and therefore the true nature of the transient eddy forcing is hidden. The feedbacks on the time-mean externally forced response is the sum of the long-term feedbacks (28) plus $D$ times the transient feedback (27):

$$
\begin{pmatrix}
    b_{11} + Da_1 & b_{12} \\
    b_{21} & b_{22} + Da_2
\end{pmatrix} = \begin{pmatrix}
    0.0419 & 0.0667 \\
    -0.0572 & -0.1180
\end{pmatrix},
$$  

(29)

where recall we assumed only the diagonal elements of the transient feedback are nonzero in (27). Unlike $b$, the total feedback in (29) is strongly negative for EOF2 and significantly weaker, although still positive for EOF1. The strong negative feedback for EOF2 is consistent with Linear Response Function (LRF) experiments (Hassanzadeh and Kuang (2016)) where we have imposed EOF2 $\bar{u}$ anomalies with external forcing. The eddies act strongly to damp the imposed anomalies (not shown), which is consistent with (29) but not a feedback analysis that ignores the transient feedback (i.e. Lorenz and Hartmann (2001); Lubis and Hassanzadeh (2021)). This discrepancy also suggests that Fluctuation Dissipation Theorem (FDT) results based on a state vector consisting of only $\bar{u}$ anomalies may give incorrect results. Because the transient feedback equation, (13), is prognostic instead of diagnostic, eddy fields should also be included in the FDT state vector. How exactly this should be done is not clear since the eddy forcing includes a stochastic component and this is a subject of future research.
5. Discussion and Conclusions

In this paper we have developed a theory for the transient negative response to $\bar{u}$ anomalies. This response lasts about 5-7 days and is ubiquitous: it occurs for any $\bar{u}$ not just EOF1. Initial value experiments demonstrate that the transient negative eddy forcing is a response to $\bar{u}$ anomalies and therefore represents a negative wave-mean-flow feedback. To operate, all that is required is a pre-existing eddy field. The pre-existing eddy field could be forced by baroclinic instability as in observations or a GCM, or it could be forced explicitly as in a barotropic model. If the background $\bar{u}$ is changed, the eddy field adjusts to the new mean state in such a way that the long-wave eddy forcing damps the $\bar{u}$ anomalies and the short-wave eddy forcing reinforces the $\bar{u}$ anomalies. The long-wave response is much larger than the short-wave response, hence the total effect is damping. This dependence on eddy wave length is consistent with observations (Lorenz and Hartmann (2001)) and in GCMs.

We also develop an analytic model of the transient feedback which shows that the sign of the response depends on the relative role of advection by $\bar{u}$ versus Rossby wave retrogression on the phase lines of the pre-existing Rossby wave. For short waves, advection dominates and so phase lines “tilt” with the shear of the imposed $\bar{u}$ anomaly, which implies reinforcing eddy momentum fluxes via the kinematics of the eddy streamfunction. For long waves, Rossby wave retrogression dominates. Because the absolute vorticity gradient includes the negative of the second derivative of the zonal wind, $-\bar{u}_{yy}$, positive background vorticity gradient anomalies are colocated with positive $\bar{u}$ anomalies. When retrogression dominates, increases in vorticity gradients imply the waves tilt against the shear, which means the eddy momentum fluxes damp the $\bar{u}$ anomalies. Our analytic model of the transient feedback also correctly predicts that the long-wave negative response is about an order of magnitude greater than the short-wave positive response, hence the total transient response is strongly negative.

Experiments with a barotropic model also demonstrate that the transient response is essentially independent of the mean state. For example, an eddy field taken from the GCM gives the same eddy forcing response to a change in $\bar{u}$ regardless of whether the background $\bar{u}$ is taken from the GCM or whether the background $\bar{u}$ is zero everywhere.

Via the mathematics relating the auto-covariance and the power spectrum, the negative transient response is synonymous with quasi-periodicity in the eddy forcing. This quasi-periodicity has
striking similarities with the quasi-periodicity in the eddy heat flux and eddy kinetic energy associated with BAM (Thompson and Woodworth (2014)). For example, Zurita-Gotor (2017) find that the quasi-periodicity in the heat flux is due to a reduction in power at the low frequencies that is the result of anti-correlation between the heat flux of different zonal wavenumbers. This same anti-correlation is the source of the eddy forcing periodicity as well (Fig. 9). Furthermore, GCM simulations with a fixed zonal-mean state show that the quasi-periodicity is mediated by the zonal-mean in both eddy forcing and eddy heat flux.

The analytic model also shows that the forcing of the transient eddy response is proportional to \( \frac{du}{dt} + Du \), where \( D \) is the Rayleigh damping constant. The unique form of the forcing of the transient feedback leads to what seems like a contradiction regarding its impacts: for internal variability the transient feedback is limited to short time lags, but for the time-mean response to external forcing the “transient feedback” impacts the response about as much as the long-term feedback. The apparent contradiction is resolved by realizing that \( \frac{du}{dt} \) and \( Du \) cancel or nearly cancel for decaying variability but the \( \frac{du}{dt} \) is identically zero for the time-mean. In addition, the fact that the transient feedback involves a prognostic rather than diagnostic equation suggests that FDT predictions based on a \( \bar{u} \) state vector are incomplete. Instead, eddy fields should be included in the state vector of FDT.

Given the relationship between eddy forcing and meridional wave activity flux, the transient negative response acts as a mechanism for trapping wave activity: positive eddy forcing anomalies, which are associated with waves leaving a location, are followed by negative eddy forcing anomalies, which are associated with waves returning. This mechanism is independent of the traditional waveguide caused by Rossby wave turning latitudes (e.g. Hoskins and Ambrizzi (1993)), instead the interaction between a wave and a time varying background flow lead to trapping of wave activity. The effect of background flow transience on the trapping of Rossby waves has been studied by Keller and Veronis (1969), Pandolfo and Sutera (1991) and Monahan and Pandolfo (2001). This current work is unique because the trapping occurs via background flow changes caused by the Rossby wave itself.
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Data availability statement. Please contact the author for the model code and data.

APPENDIX A

Analytic Solution of Transient Feedback

a. Perturbation Vorticity Equation

In this appendix, we derive the analytic solution of the transient feedback based on the approximation in (4) applied to the linearized barotropic vorticity equation. The assumptions in this appendix are identical to the assumptions in 4b. For example, the background zonal-mean zonal wind is constant in $y$ with value $U - c$, the background eddy vorticity is given by (7) and the perturbed background flow takes a simple sinusoidal form (11). The vorticity gradient associated with (11) is:

$$-\Delta \vec{u}_{yy} = n^2 u(t) \cos(ny). \quad (A1)$$

Substituting (11) and (A1) in (8) and using the function form of the background eddy vorticity, (7), to simplify the derivatives, we get

$$\hat{L}(\zeta') = u(t) \cos(ny)ik \left(1 - \frac{n^2}{k^2}\right) \zeta \exp(ikx). \quad (A2)$$

The negative of (A2) is the forcing on the right hand side of (4).

Next we consider the $L(\hat{\zeta'})$ term on the left hand side of (4). Because the unperturbed background flow is uniform, the operator $L$ simply multiplies $\hat{\zeta'}$ by a complex constant. Therefore the perturbation eddy vorticity ($\hat{\zeta'}$) on the left hand side of (4) simply “inherits” the $\cos(ny) \exp(ikx)$ spatial dependence factor from the right hand side (i.e. (A2)):

$$\zeta' = \hat{\zeta}(t) \cos(ny) \exp(ikx), \quad (A3)$$
where $\zeta(t)$ is a complex function of time. Now that the form of $\hat{\zeta}'$ is known, we see that

$$L(\hat{\zeta}') = ik \left( U - c - \frac{\beta}{k^2 + n^2} \right) \hat{\zeta}(t) \cos(ny) \exp(ikx),$$  \hspace{1cm} (A4)$$

where we use the fact that $\partial \nabla^2/\partial x = -ik/(k^2 + n^2)$ for waves with meridional wavenumber $n$. Substituting (A2), (A3) and (A4) in (4) and cancelling the $\cos(ny) \exp(ikx)$ factor:

$$\frac{d\hat{\zeta}}{dt} + \left( ik \left( U - c - \frac{\beta}{k^2 + n^2} \right) + D \right) \hat{\zeta} = -ik \left( 1 - \frac{n^2}{k^2} \right) u(t) \zeta.$$  \hspace{1cm} (A5)$$

Equation (A5) completely describes the solution, however, it is difficult to understand the eddy forcing (i.e. eddy momentum flux convergence) produced by this system. In the next section, we derive the eddy forcing resulting from (A5).

**b. Derivation of the Eddy Momentum Flux Convergence Equation**

In this section, we develop an equation for the eddy momentum flux convergence response to changing background zonal winds of the form (11). First, for a non-divergent flow, the eddy momentum flux convergence $(-\partial \overline{uv}/\partial y)$ is identical to the eddy vorticity flux $(\overline{\zeta'v'})$. Under the assumption that terms quadratic in perturbations are neglected, the perturbed eddy momentum flux convergence is:

$$\overline{(\zeta' + \hat{\zeta}') (v' + \hat{v}') - \overline{\zeta'v'}} = \overline{\zeta'v'} + \overline{\hat{\zeta}'\hat{v}'}.$$  \hspace{1cm} (A6)$$

Next, we write the eddy vorticity flux in terms of the complex coefficients multiplying $\exp(ikx)$:

$$\overline{\zeta'v'} + \overline{\hat{\zeta}'\hat{v}'} = \frac{1}{2} \cos(ny) \Re \left( (\hat{\zeta}'v^* + \zeta'\hat{v}^*) \right),$$  \hspace{1cm} (A7)$$

where the unprimed quantities denote complex coefficients with no $y$ dependence, $\hat{\zeta}$ and $\zeta$ are given by (A3) and (7) respectively, $v^*$ denotes the complex conjugate of $v$ and we use the fact that the zonal mean of two waves of the form $\exp(ikx)$ with complex amplitudes $x$ and $y$ is $\Re(xy^*/2)$. For the remainder of the derivation, we ignore the factor of $1/2$ because the derivation below will lead to the same $1/2$ factor in front of the “effective forcing”, leading to cancelation. The $y$ dependence of the momentum flux convergence, (A7), is the same as the zonal wind perturbation, (11), so under the above approximations the eddy forcing either reinforces or damps the zonal wind anomalies.
but does not shift them in latitude. Therefore the solution can be completely characterized by the eddy forcing in phase with the zonal wind, which we call \( m \):

\[
m = \Re \left( (\hat{\zeta} v^* + \zeta \hat{v}^*) \right). \tag{A8}
\]

For simplicity, we do not include “hat” on the perturbation eddy forcing, \( m \).

Recall that the unperturbed wave is assumed to be independent of \( y \) (see Section 4), therefore:

\[
v = -i \frac{\zeta}{k}. \tag{A9}
\]

The perturbed wave is sinusoidal in \( y \), (A3), therefore:

\[
\hat{v} = -i \frac{k \hat{\zeta}}{k^2 + n^2}. \tag{A10}
\]

Substituting (A9) and (A10) into (A8):

\[
m = \frac{1}{k} \Re \left( i \hat{\zeta} \zeta^* + \frac{k}{k^2 + n^2} \Re \left( i \zeta \hat{\zeta}^* \right) \right) \tag{A11}
\]

\[
= -\frac{1}{k} \Im \left( \hat{\zeta} \zeta^* \right) - \frac{k}{k^2 + n^2} \Im \left( \zeta \hat{\zeta}^* \right) \tag{A12}
\]

\[
= -\frac{1}{k} \Im \left( \hat{\zeta} \zeta^* \right) + \frac{k}{k^2 + n^2} \Im \left( \zeta \hat{\zeta}^* \right) \tag{A13}
\]

\[
m = -\frac{kn^2}{k^2(k^2 + n^2)} \Im \left( \hat{\zeta} \zeta^* \right), \tag{A14}
\]

where (A11) to (A12) uses the fact that \( \Re \left( i \hat{\zeta} \zeta^* \right) = -\Im \left( \hat{\zeta} \zeta^* \right) \) and (A12) to (A13) uses the fact that \( \Im \left( \hat{\zeta} \zeta^* \right) = -\Im \left( \zeta \hat{\zeta}^* \right) \). We define \( r \) to be the corresponding real part of (A14):

\[
r = \frac{-kn^2}{k^2(k^2 + n^2)} \Re \left( \hat{\zeta} \zeta^* \right). \tag{A15}
\]

To use (A14), we next multiply (A5) by \( \zeta^* \):

\[
\frac{d \hat{\zeta} \zeta^*}{dt} + \left( ik \left( U - c - \frac{\beta}{k^2 + n^2} \right) + D \right) \hat{\zeta} \zeta^* = -ik \left( 1 - \frac{n^2}{k^2} \right) u \zeta \zeta^*, \tag{A16}
\]

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where we use the fact that $\zeta^*$ does not depend on time to move it inside the time derivative.

Multiplying (A16) by 

$$\frac{-kn^2}{k^2(k^2+n^2)}$$  \hspace{1cm} (A17)

and substituting (A14) and (A15):

$$\frac{d(r + im)}{dt} + \left(ik\left(U - c - \frac{\beta}{k^2+n^2}\right) + D\right)(r + im) = -\frac{n^2(k^2-n^2)}{k^2(k^2+n^2)}u\zeta^*,$$ \hspace{1cm} (A18)

where $\zeta^* = \zeta^*$ is the eddy vorticity variance in the mean state. Separating the real and imaginary parts of (A18):

$$\frac{dr}{dt} + Dr - k\left(U - c - \frac{\beta}{k^2+n^2}\right)m = 0,$$ \hspace{1cm} (A19)

$$\frac{dm}{dt} + Dm + k\left(U - c - \frac{\beta}{k^2+n^2}\right)r = \frac{n^2(k^2-n^2)}{k^2(k^2+n^2)}u\zeta^*.$$ \hspace{1cm} (A20)

Finally, operating on (A20) with $\frac{d}{dt} + D$ and substituting (A19) gives (13).

**APPENDIX B**

**Derivation of Methodology for Estimating Feedback Parameters**

In this Appendix we describe the method for estimating feedback parameters from lagged covariances using (25) and (26). First, to simplify the notation, we define:

$$\tilde{D}_{jk}(l) = \sum_{s=0}^{\infty} w(s) D_{jk}(l-s)$$  \hspace{1cm} (B1)

$$\tilde{U}_{jk}(l) = \sum_{s=0}^{\infty} w(s) U_{jk}(l-s).$$  \hspace{1cm} (B2)

With this simplification, minimizing the error in (25) and (26) leads to the following expression to minimize over time lags, $l$, is

$$\min \left(\frac{1}{2} \sum_{l=1}^{l_2} \sum_{j=1}^{2} \left(a_1 \tilde{D}_{1j} + b_{11} \tilde{U}_{1j} + b_{12} \tilde{U}_{2j} - M_{1j}\right)^2\right),$$ \hspace{1cm} (B3)
where $l_1$ and $l_2$ are the range of lags to minimize the error. Setting the derivatives of (B3) with respect to $a_1$, $b_{11}$ and $b_{12}$ equal to zero, we get a system of three equations and three unknowns, which can be easily solved to get $a_1$, $b_{11}$ and $b_{12}$:

$$
\begin{pmatrix}
\sum_{j,l} M_{1j}(l) \tilde{D}_{1j}(l) \\
\sum_{j,l} M_{1j}(l) \tilde{U}_{1j}(l) \\
\sum_{j,l} M_{1j}(l) \tilde{U}_{2j}(l)
\end{pmatrix}
= \begin{pmatrix}
\sum_{j,l} (\tilde{D}_{1j}(l))^2 \\
\sum_{j,l} (\tilde{U}_{1j}(l))^2 \\
\sum_{j,l} (\tilde{U}_{2j}(l))^2
\end{pmatrix}
\begin{pmatrix}
a_1 \\
b_{11} \\
b_{12}
\end{pmatrix}.
$$

(B4)

The solution for the coefficients $a_2$, $b_{22}$ and $b_{21}$ are the same except the 1 subscripts are replaced with 2 and the 2 subscripts are replaced with 1 in (B4).

References


